Scaling of Anisotropic Flows and Nuclear Equation of State in Intermediate Energy Heavy Ion Collisions*

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(Dated: February 1, 2008)

Elliptic flow (v_2) and hexadecupole flow (v_4) of light clusters have been studied in details for 25 MeV/nucleon ^{86}Kr + ^{124}Sn at large impact parameters by Quantum Molecular Dynamics model with different potential parameters. Four parameter sets which include soft or hard equation of state (EOS) with/without symmetry energy term are used. Both number-of-nucleon (A) scaling of the elliptic flow versus transverse momentum (p_t) and the scaling of v_4/A^2 versus $(p_t/A)^2$ have been demonstrated for the light clusters in all above calculation conditions. It was also found that the ratio of $v_4/{v_2}^2$ keeps a constant of 1/2 which is independent of p_t for all the light fragments. By comparisons among different combinations of EOS and symmetry potential term, the results show that the above scaling behaviors are solid which do not depend the details of potential, while the strength of flows is sensitive to EOS and symmetry potential term.

Keywords: Anisotropic flow, number-of-nucleon scaling, EOS, symmetry energy
PACC: 2410, 2570, 2587

Anisotropic flows are very useful to explore heavy-ion collision dynamics since it results from the transition of the original space-time asymmetry into a momentum space anisotropy for the non-central collision [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14]. Different mechanisms will contribute the final momentum anisotropy, i.e. flow. Many studies of the dependence of the directed flow (v_1) and the elliptic flow (v_2) on beam energies, mass number, isospin and impact parameter have been carried out and much interesting physics has been demonstrated on the properties and origin of the collective motion in both nucleonic or partonic levels. Very recently, we carried out a Quantum Molecular Dynamics model calculation with hard equation of state and symmetry energy term and found that there is a nucleon-number scaling for the elliptic flow of light particles up to the mass number A = 4 [15]. In this work, we shall present more details for the nucleon number dependence of the anisotropic flows v_2 and v_4 for ^{86}Kr + ^{124}Sn collisions at 25 MeV/nucleon and large impact parameters ($b = 7-10fm$) with different EOS and symmetry energy interaction. The scaling behaviors look robust since they do not depend the parameters used in the model, and the sensitivities of EOS and symmetry potential for the v_2 and v_4 are discussed.

Anisotropic flow is defined as the different n-th harmonic coefficient v_n of the Fourier expansion for the particle invariant azimuthal distribution

$$\frac{dN}{d\phi} \propto 1 + 2\sum_{n=1}^{\infty} v_n \cos(n\phi), \tag{1}$$

where ϕ is the azimuthal angle between the transverse momentum of the particle and the reaction plane. Note that the z-axis is defined as the direction along the beam and the impact parameter axis is labelled as x-axis. The first harmonic coefficient v_1 represents directed flow, $v_1 = \langle cos\phi \rangle = \langle \frac{p_x}{p_t} \rangle$, where $p_t = \sqrt{p_x^2 + p_y^2}$ is transverse momentum. While the v_2 which measures the eccentricity of the particle distribution in the momentum space represents elliptic flow,

$$v_2 = \langle cos(2\phi) \rangle = \langle \frac{p_x^2 - p_y^2}{p_t^2} \rangle, \tag{2}$$

and v_4 represents the 4-th momentum anisotropy, namely hexadecupole flow:

$$v_4 = \left\langle \frac{p_x^4 - 6p_x^2 p_y^2 + p_y^4}{p_t^4} \right\rangle. \tag{3}$$

The intermediate energy heavy-ion collision dynamics is complex since both mean field and nucleon-nucleon collisions are playing the competition roles. Furthermore, the isospin dependent role should be also incorporated for asymmetric reaction systems. Isospin dependent Quantum Molecular Dynamics model (IDQMD) has been affiliated with isospin degrees of freedom with mean field and nucleon-nucleon collision [16, 17, 18, 19, 20, 21, 22, 23]. The IDQMD model can explicitly represent the many body state of the system and principally contain correlation effects to all orders and all fluctuations, and can

*Supported partially by the National Natural Science Foundation of China under Grant No 10535010 and 10610285, the Shanghai Development Foundation for Science and Technology under Grant Numbers 06JC14082 and 05XD14021, and CAS project KJCX3.SYW.N2
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describe the time evolution of the colliding system well. When the spatial distance (Δr) is closer than 3.5 fm and the momentum difference (Δp) is smaller than 300 MeV/c between two nucleons, two nucleons can coalesce into a cluster [16]. With this simple coalescence mechanism which has been extensively applied in transport theory, different size clusters can be recognized.

In the model the nuclear mean-field potential is parameterized as

$$U(\rho,\tau_z) = \alpha(\frac{\rho}{\rho_0}) + \beta(\frac{\rho}{\rho_0})^{\gamma} + \frac{1}{2}(1-\tau_z)V_c$$

$$+ C_{sym}\frac{(\rho_n - \rho_p)}{\rho_0}\tau_z + U^{Yuk} \qquad (4)$$

where ρ_0 is the normal nuclear matter density ($0.16 fm^{-3}$), ρ_n, ρ_p and ρ are the neutron, proton and total densities, respectively. τ_z is z-th component of the isospin degree of freedom, which equals 1 or -1 for neutrons or protons, respectively. The coefficients α, β and γ are parameters for nuclear equation of state. C_{sym} is the symmetry energy strength due to the density difference of neutrons and protons in nuclear medium, which is important for asymmetry nuclear matter [24, 25, 26] (here C_{sym} = 32 MeV is used to consider symmetry energy effect or isospin-dependent potential, and $C_{sym} = 0$ for no symmetry energy effect or isospin-independent potential). V_c is the Coulomb potential and U^{Yuk} is Yukawa (surface) potential. In this work, we take $\alpha = 124$ MeV, β = 70.5 MeV and $\gamma = 2$ which corresponds to the so-called hard EOS with an incompressibility of K = 380 MeV, and α = -356 MeV, β = 303 MeV and $\gamma = 7/6$ which corresponds to the so-called soft EOS with an incompressibility of K = 200 MeV. In the present study, four combinations with different potential parameters, i.e. parameters of hard or soft EOS with or without symmetry energy effect (i.e. C_{sym} = 32 or 0 MeV), for the collision system of ^{86}Kr + ^{124}Sn at 25 MeV/nucleon with impact parameter from 7 fm to 10 fm were carried out. The physics results were extracted at the time of 200 fm/c when the system has been in the freeze-out stage.

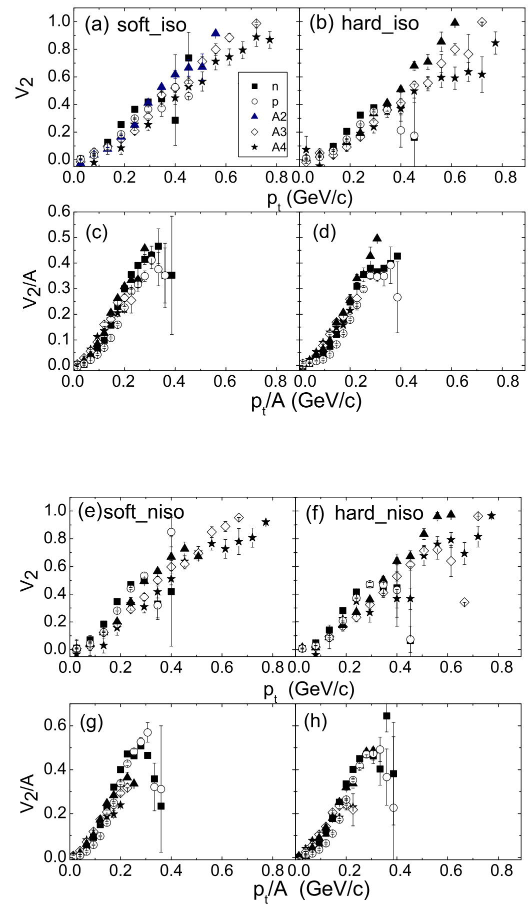

FIG. 1: (a), (b), (e) and (f): Elliptic flow as a function of transverse momentum (p_t) for the simulation with different parameters of EOS with or without symmetry energy term. (a) for soft EOS with symmetry potential (*soft_iso*); (b) for hard EOS with symmetry potential (*hard_iso*); (e) for soft EOS without symmetry potential (*soft_niso*); and (f) for hard EOS without symmetry potential (*hard_niso*). Squares represent for neutrons, circles for protons, triangles for fragments of A=2, diamonds for A=3 and stars for A=4. Fig.(c), (d), (g) and (h) presents nucleon-number normalized elliptic flow as a function of transverse momentum per nucleon corresponding to the case of (a), (b), (e) and (f), respectively.

The Fig.1 (a), (b), (e) and (f) shows transverse momentum dependence of elliptic flows for mid-rapidity light fragments in four different calculation conditions: (a) for soft EOS with symmetry potential (*soft_iso*); (b) for hard EOS with symmetry potential (*hard_iso*); (e) for soft EOS without symmetry potential (*soft_niso*) and (f) for hard EOS without symmetry potential (*hard_niso*). In all cases, elliptic flow is positive and it increases with the increasing p_t, which is apparently similar to RHIC's results [12, 13]. Of course, the mechanism is very different. In intermediate energy domain, collective rotation is one of the main mechanisms to induce the positive elliptic flow [2, 3, 27, 28, 29]. However, at RHIC energies it is the strong pressure which is built in early initial almond anisotropy of the geometrical overlap zone between both colliding nuclei that drives the positive elliptic flow [12]. The corresponding nucleon-number scaled elliptic flows are plotted in Fig.1 (c), (d), (g) and (h) as a function of transverse momentum per nucleon. From these panels, it seems that the number of nucleon scaling for elliptic flow exists for light fragments at low p_t/A ($p_t/A < 0.2GeV/c$). This behavior is apparently similar to the number of constituent quarks scaling of elliptic flow versus transverse momentum per constituent quark (p_t/n) for different mesons and baryons which was observed at RHIC [12]. Since all calculations show the similar scaling behavior, this scaling behavior is robust, and it is independent of the details of EOS and symmetry potential.

To quantitatively look the difference of the flows in different calculation conditions, we compare the values of v_2/A for the four simulation conditions (see Fig.2). The figures show that the difference between different

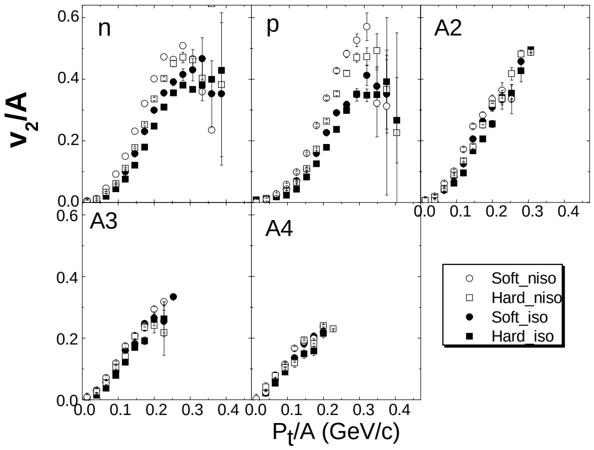

FIG. 2: Comparisons of the values of v_2/A versus p_t/A in the different simulation conditions. The meanings of the symbols are depicted in right bottom corner.

simulations is big for neutrons and protons but a little small for the fragments of $A = 2$, $A = 3$ and $A = 4$. The reason is that the emitted protons and neutrons can feel the role of mean field (EOS) directly, while the light fragments have weak sensitivity since they are indirected products by the coalescence mechanism in the present model. Approximately at the same p_t/A, the elliptic flow is larger for soft EOS than the one for hard EOS, and it is larger for EOS with symmetry potential than the case without symmetry potential. Considering that the symmetry potential is basically positive for the studied reaction system (more neutrons than protons), symmetry potential will make the whole EOS stiffer. In this case, we can say that the stiffer the EOS, the smaller the flow. In other words, we can say the strength of elliptic flow per nucleon is sensitive to the EOS and symmetry potential.

So far, there is rare studies about higher order flows, such as v_4, experimentally and theoretically in this energy domain. Here we try to explore the behavior of v_4. First we draw v_4/A as a function of p_t/A mimicing the behavior of elliptic flow (see (a), (b), (e) and (f) of Fig.3) for four different calculation conditions. It shows that v_4/A is positive and increases with p_t/A, but there seems no simple scaling behavior as v_2 shows. Considering that RHIC experimental data have demonstrated that a scaling relation among hadron anisotropic flows holds, i.e., $v_n(p_t) \sim v_2^{n/2}(p_t)$ [30], we plot v_4/A^2 as a function of $(p_t/A)^2$ in Fig. 3 (c), (d), (g) and (h) for the corresponding calculation conditions of Fig.3(a), (b), (e) and (f). Now the points of different light fragments nearly merge together at low $(p_t/A)^2$, which means a certain of scaling law holds between two variables. All the calculation cases show that there is the scaling behavior for v_4/A^2 versus $(p_t/A)^2$, and this behavior is robust regardless the parameters which we used for EOS.

Since the above scaling behavior assumes $v_n(p_t) \sim v_2^{n/2}(p_t)$, so we plot v_4/v_2^2 as a function of p_t in Fig. 4 for the four simulations. The figures show that the ratios

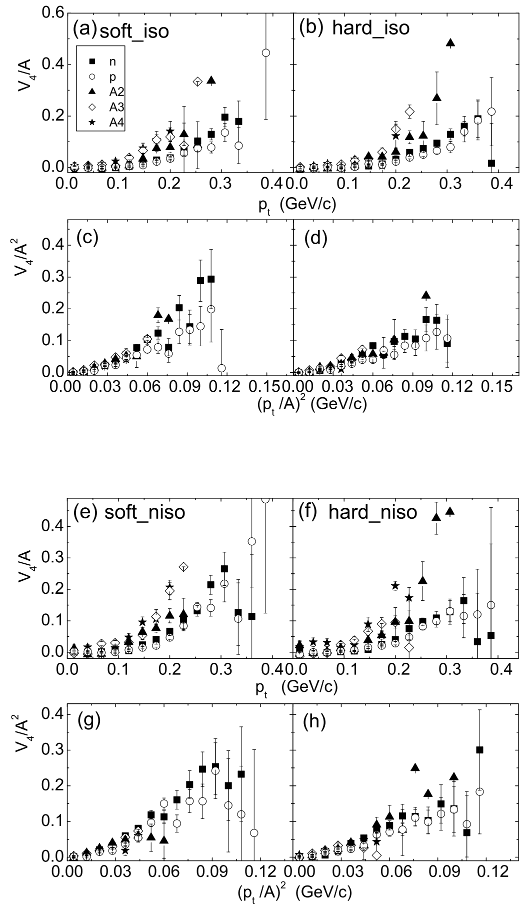

FIG. 3: Same as Fig.1 but for v_4/A versus p_t [(a), (b), (e) and (f)] and v_4/A^2 versus $(p_t/A)^2$ [(c), (d), (g) and (h)].

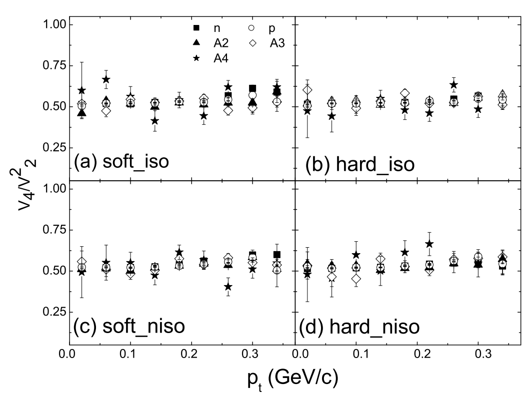

FIG. 4: The ratios of v_4/v_2^2 for neutrons (squares), proton (circles), the fragments of $A = 2$ (triangles), $A = 3$ (diamonds) and $A = 4$ (stars) versus p_t for the four simulations in different calculation conditions.

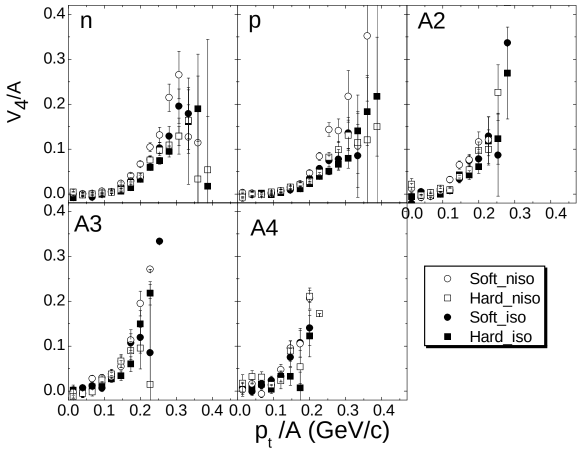

FIG. 5: Comparison of the values of v_4/A versus p_t/A. The meanings of the symbols are depicted in right bottom corner.

of v_4/v_2^2 for different fragments up to $A = 4$ are about a constant of 1/2 in all simulation cases. Because v_2/A can be scaled with p_t/A, v_4/A^2 should scale versus $(p_t/A)^2$, which is exactly what we see in Fig. 4. One point is worth to be mentioned comparing to the RHIC studies where the data shows $v_4/v_2^2 \sim 1.2$ [30], $v_4/v_2^2 \sim 1/2$ for the light nuclear fragments in this nucleonic level coalescence mechanism rather than the value of 3/4 for mesons or 2/3 for baryons in quark coalescence model [31]. Coincidentally, the predicted value of the ratio of v_4/v_2^2 for hadrons is also 1/2 if the matter produced in ultra-relativistic heavy ion collisions reaches to thermal equilibrium and its subsequent evolution follows the laws of ideal fluid dynamics [32]. It is interesting to note the same ratio was predicted in two different models at very different energies, which is of course worth to be further investigated in near future. One possible interpretation is that the big nucleon-nucleon cross sections in low energy HIC make the system to reach thermal equilibrium and may induce the fluid-like behavior of nuclear medium before the light fragments are coalesced by nucleons. In this case, the value of v_4/v_2^2 of light fragments could be $\sim 1/2$ as Ref.[32] shows.

The values of v_4/A versus p_t/A with different simulation parameters are also presented for light fragments, see Fig.5. The figures are similar to those in Fig.2, and the effects of EOS and symmetry potential on v_4/A are also similar to their effects on v_2. However, comparing with the v_2's sensitivity to the EOS and symmetry potential, v_4/A is not so salient.

To summarize, we investigated the behavior of anisotropic flows as a function of transverse momentum for light fragments for the simulations of 25 MeV/nucleon $^{86}Kr+^{124}Sn$ collisions in peripheral collisions by IDQMD model in the potential parameters of hard or soft EOS with or without symmetry energy term. It was found that for all the four type simulations v_2 and v_4 of light fragments are positive and increase with p_t/A. When we plot v_2 per nucleon (v_2/A) versus p_t/A for all light particles, all curves collapse onto the same curve. Similarly, the values of v_4/A^2 merge together as a function of $(p_t/A)^2$ for all light particles. Furthermore, it was found that v_4 can be well scaled by v_2^2, and the value of $v_4/v_2^2 \sim 1/2$ which does not depend on transverse momentum. The above scaling behaviors can be seen as an outcome of the nucleonic coalescence, and it illustrates that the number-of-nucleon scaling for elliptic flow exists in intermediate energy heavy ion collision. In addition, the values of v_2/A and v_4/A were compared in different simulation conditions, and it was shown that the values of the v_2 are sensitive to the EOS and symmetry potential, especially for neutrons and protons.

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